## Coulomb's Law Video Main Points

We watched the quirky but classic video "Coulomb's Law." In this video, the main man, Princeton University professor Eric Rogers, investigated the force experienced by electrically charged objects. Of course, as we already know, charged objects create fields around themselves, and these fields exert forces on other charged objects that are within them.

By charging two balls and varying the distance between them, he was able to find out how this electric force depends on the separation of the charged objects. In doing this he found that doubling the distance between them cut the electric force to one quarter of its original value. When he tripled the distance between the two charged balls, he found that the force was cut to one ninth as much as its original value. Because of the fact that $F_{\text {elec }}=q \mathbb{E}$, this means that the electric field $\mathbb{E}$, which exerts the force, is inversely proportional to the square of the separation. We call this kind of relationship an "inverse square law." It is an inverse proportion, because you square the factor by which the separation changes and take its inverse in order to find the factor by which the electric field changes. For separation we use the symbol $r$ (because in physics the separation is often the radius of an orbit of an electron around an atomic nucleus.).

$$
\begin{equation*}
\mathbb{E} \propto \frac{1}{r^{2}} \tag{1}
\end{equation*}
$$

After this, Professor Rogers wanted to find out how the electric force depends on the amount of electric charge one of these balls possesses. He chose one of the balls and cut its charge in half by touching it with an identical uncharged ball, therefore sharing the charge between the two balls $50-50$. He found as a result that the force on the other charged ball (and therefore the strength of the electric field of the chosen ball) was also cut in half. He cut the charge on the chosen ball in half again, reducing the amount of its charge to only one-quarter of its original value. This cut the force on the other charged ball, and therefore the strength of the field of the chosen ball, to a quarter of its original value. From this he concluded that the electric force on the other charged ball, and therefore the strength of the electric field of the chosen ball, is directly proportional to the amount of electric charge on the chosen ball. The symbol we use for the quantity of electric charge an object has is $q$, so we write this mathematically as:

$$
\begin{equation*}
\mathbb{E} \propto q \tag{2}
\end{equation*}
$$

Combining these two proportions, Proportion (1) and Proportion (2), gives us the following proportion:

$$
\begin{equation*}
\mathbb{E} \propto \frac{q}{r^{2}} \tag{3}
\end{equation*}
$$

To make this an equation rather than a mere proportion, we need to put another factor into Proportion (3). The symbol physicists use for this factor is $k$.

$$
\begin{equation*}
\mathbb{E}=k \frac{q}{r^{2}} \tag{4}
\end{equation*}
$$

To find the numerical value of $k$ we would first have to know the values for all of the variables, $\mathbb{E}, q$, and $r$. Then we could do the algebra to solve for $k$.

Physicists have set up situations in which they actually could measure all the variables, $\mathbb{E}, q$, and $r$, and then they solved for $k$. It turns out that $k$ always has a value of $9.0 \times 10^{9} \mathrm{Nm}^{2} / \mathrm{C}^{2}$. The units for $k$ were chosen just so that $\mathbb{E}$ is in $\mathrm{N} / \mathrm{C}$, as is necessary.

In conclusion, we have Coulomb's Law:

$$
\begin{equation*}
\mathbb{E}=\left(9.0 \times 10^{9} \frac{\mathrm{~N} \cdot \mathrm{~m}^{2}}{\mathrm{C}^{2}}\right) \frac{q_{s}}{r^{2}} \tag{5}
\end{equation*}
$$

In Equation (5), $\mathbb{E}$ stands for the strength of the electric field produced by a charged object at some point in space, $r$ stands for the distance from this charged object to the point in space, and $q_{s}$ stands for the amount of electric charge this object possesses. Because it is the source of the electric field, let's label it with the subscript " $s$." We measure the amount of electric charge $q$ in Coulombs, C. A coulomb is much more electric charge than a charged piece of clothing has when you remove it from the dryer. We measure the charges on everyday objects in microcoulombs $(\mu \mathrm{C})$ and nanocoulombs ( nC ). A microcoulomb is a millionth $\left(10^{-6}\right)$ of a Coulomb. A nanocoulomb is a billionth $\left(10^{-9}\right)$ of a Coulomb.

## Example

Calculate the electric field 3 m away from a particle that has an electric charge of 0.1 C .

## Solution

$$
\begin{gathered}
\mathbb{E}=k \frac{q_{s}}{r^{2}} \\
\mathbb{E}=9 \times 10^{9} \cdot \frac{0.1}{3^{2}} \\
\mathbb{E}=\frac{9 \times 0.1}{3^{2}} \times 10^{9} \\
\mathbb{E}=0.1 \times 10^{9} \mathrm{~N} / \mathrm{C} \\
\mathbb{E}=1 \times 10^{8} \mathrm{~N} / \mathrm{C}
\end{gathered}
$$

## Problems

1. Calculate the electric field strength 2 m away from a particle that has an electric charge of 0.4 C .
2. Calculate the electric field strength 0.5 m away from a particle that has an electric charge of 0.2 C .
3. Calculate the electric field strength 0.01 m away from a particle that has an electric charge of $4 \times 10^{-6} \mathrm{C}$.
4. An electron in a hydrogen atom moves around the proton at the center of the atom. (The proton is the nucleus.) The average distance of the electron from the proton is about $5.3 \times 10^{-11} \mathrm{~m}$. The charge on a proton is $+1.6 \times 10^{-19} \mathrm{C}$. The electron has exactly the same amount of charge as the proton, but it has a negative charge instead of a positive charge. Calculate the strength of the electric field created by the proton out where the electron orbits.

## Gravitational Force and Electric Force

Last semester we found that we could calculate the amount of gravitational force exerted on an object that has mass and happens to be in the gravitational field created by another object with mass. We use the symbol $m$ for the mass of the object that feels the force. The symbol $g$ stands for the strength of the gravitational field. Here's how we calculated the force experienced by the object:

$$
\begin{equation*}
F_{\text {grav }}=m g \tag{6}
\end{equation*}
$$

At the surface of Earth we found that $g=9.8 \mathrm{~N} / \mathrm{kg}$.

In the same way, we can calculate the electric force exerted on an object that has electric charge and happens to be in the electric field created by another electrically charged object.

$$
\begin{equation*}
F_{\text {elec }}=q \mathbb{E} \tag{7}
\end{equation*}
$$

Compare the previous two equations. They follow the same pattern. Just as $q$ stands for the amount of electric charge an object has, $m$ stands for how much "gravitational charge" an object has. We use the word "mass" instead of calling it "gravitational charge," but you should get the idea. We measure the amount of electric charge an object possesses in Coulombs (C). We measure the amount of gravitational charge (mass) an object has in kilograms (kg).

Also, we know that $g$ is the strength of a gravitational field in $\mathrm{N} / \mathrm{kg}$. In just the same way $\mathbb{E}$ is the strength of an electric field in $\mathrm{N} / \mathrm{C}$.

## Example 1

You find that the strength of an electric field is $2000 \mathrm{~N} / \mathrm{C}$. How much force will it exert on a particle that possesses 0.004 C ?

## Solution

$$
\begin{gathered}
F_{\text {elec }}=q \mathbb{E} \\
F_{\text {elec }}=0.004 \cdot 2000 \\
F_{\text {elec }}=8 \mathrm{~N}
\end{gathered}
$$

## Example 2

A dust particle carries an electric charge of $2 \times 10^{-4} \mathrm{C}$. It is in an electric field that exerts a force of 4 N on it. What is the strength of this electric field?

Solution

$$
\begin{aligned}
& F_{\text {elec }}=q \mathbb{E} \\
& \mathbb{E}=\frac{F_{\text {elec }}}{q}
\end{aligned}
$$

$$
\begin{gathered}
\mathbb{E}=\frac{4}{2 \times 10^{-4}} \\
\mathbb{E}=\frac{2}{10^{-4}} \\
\mathbb{E}=2 \times 10^{4} \mathrm{~N} / \mathrm{C}
\end{gathered}
$$

## Problems

1. How much force does an electric field with a strength of $3 \mathrm{~N} / \mathrm{C}$ exert on an object that has a charge of 2 C ?
2. An electric field has a strength of $200 \mathrm{~N} / \mathrm{C}$. How much force would it exert on an object having a charge of 0.5 C ?
3. A particle carries an electric charge of 0.004 C . It is in an electric field with a strength of $1000 \mathrm{~N} / \mathrm{C}$. How big is the electric force on this particle?
